Test questions: **Riemann Sums**

This test covers left, right, and midpoint Riemann sums and the Trapezoidal Rule

Calculator optional

1. Selected values of a function, f, are given in the table below.

x	1	4	5	7	8	12
$f(x)$	6	8	9	12	13	15

   a. Give the left-side Riemann sum with 5 subintervals for f on the interval [1, 12].

   b. Give the right-side Riemann sum with 5 subintervals for f on the interval [1, 12].

   c. Give the Trapezoidal Rule approximations with 6 subintervals for f on the interval [1, 12].

   d. Give the midpoint approximation with two subintervals for f on the interval [1, 12].

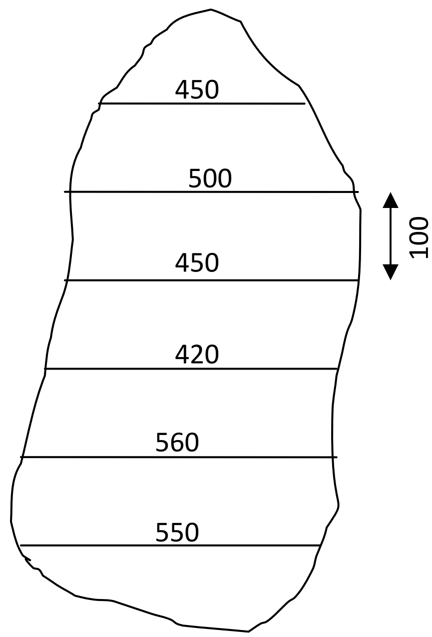
2. The function $g(x)$ is, positive, decreasing and concave up on the interval $[a, b]$. Let A be the area of the region between this function and the x-axis, let L be a left-side Riemann sum approximation of the area with n subintervals, let R be a right-side Riemann sum approximation of the area with n subintervals, and let T be a Trapezoidal approximation of the area with n subintervals. List A, L, R, and T in order from *smallest to largest.* Use a sketch to explain why your answer is correct.

3. Calculate the left-side Riemann sum for the function $h(x) = \sin(x)$ on the interval $\left[0, \frac{\pi}{2}\right]$ using these partition values: $\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$.

4.

a. The sketch at the right shows a lake that is 700 feet long from top to bottom. The width of the lake is measured at 100 foot intervals; these measurements, in feet, are marked on the sketch. (Not to scale)

b. Approximate the area of the surface of the lake. Include units of measure.



5. A scientist wants to estimate the area between the x-axis and the graph of $y = x^3 - 9x^2 + 15x + 30$ for $-1 \le x \le 7$. For safety reasons she wants to underestimate the area. She decides to use left-side and right-side Riemann sums to make the estimate. On what sub-interval(s) should she use the left-side sums and on what interval(s) should she use the right-sides sums? Justify your answer.

Answers

1. **[3 points each part, subtract 1 point for any wrong term (up to 3). If the student chooses to add the terms (which is *not* necessary on the AP exams), deduct 1point if the sum is wrong.]**

   a. Left $= 6(3)+8(1)+9(2)+12(1)+13(4)=108$

   b. Right $= 8(3)+9(1)+12(2)+13(1)+15(4)=130$

   c. Trapezoid
   $=\tfrac{1}{2}(6+8)(3)+\tfrac{1}{2}(8+9)(1)+\tfrac{1}{2}(9+12)(2)+\tfrac{1}{2}(12+13)(1)+\tfrac{1}{2}(13+15)(4)=119$

   Or Trapezoid = $\dfrac{L+R}{2}=\dfrac{108+130}{2}=119$

   d. Midpoint $=8(4)+13(5)=97$

2. **[3 points]** $R<A<T<L$ Because all right rectangles lie below the graph and all trapezoids and left rectangle lie above.

   Sketch **[2 points]**

3. **[5 points Deduct 1 point for each wrong trig value or wrong term. The first line is enough for full credit; if the student goes further, deduct a maximum of one point for all subsequent mistakes.]**

$$\begin{aligned}\text{Left} &= 0\left(\tfrac{\pi}{6}\right)+\tfrac{1}{2}\left(\tfrac{\pi}{4}-\tfrac{\pi}{6}\right)+\tfrac{\sqrt{2}}{2}\left(\tfrac{\pi}{3}-\tfrac{\pi}{4}\right)+\tfrac{\sqrt{3}}{2}\left(\tfrac{\pi}{2}-\tfrac{\pi}{3}\right)\\ &= \tfrac{1}{2}\left(\tfrac{\pi}{12}\right)+\tfrac{\sqrt{2}}{2}\left(\tfrac{\pi}{12}\right)+\tfrac{\sqrt{3}}{2}\left(\tfrac{\pi}{6}\right)\\ &\approx 0.769\end{aligned}$$

4. **[4 points for the computation; 1 point units.]**

   Using trapezoids and triangles (at top and bottom) the area is approximately

$$\begin{aligned}&=\tfrac{1}{2}(100)(450)+\tfrac{1}{2}(100)(450+500)+\tfrac{1}{2}(100)(500+450)\\ &\qquad+\tfrac{1}{2}(100)(450+420)+\tfrac{1}{2}(100)(420+560)+\tfrac{1}{2}(100)(560+550)+\tfrac{1}{2}(100)(550)\\ &=293{,}000 \text{ square feet}\end{aligned}$$

5. **[5 points: 1 point for derivative, 1 point for finding critical points, 1 point for each interval with reason.]**

Left-side for [–1, 1] and [5, 7], because the derivative is positive on these intervals, the function is increasing and the left-side sums will underestimate the value.

Right-side for [1, 5], because the derivative is negative here, the function decreases and right-side sums will underestimate the value.

$$y'(x) = 3x^2 - 18x + 15 = 0$$
$$= 3(x-1)(x-5) = 0$$

$x = 1$ and $x = 5$ are the critical points.

[30 points in all]